tivity to $T=0$ yields a finite value but the mobility of the charge carriers seems to be limited by the exchange interaction of the randomly distributed impurities. It seems unlikely that the mobility can be described by a conventional scattering theory. It is precisely in this region and at lower concentrations that the anomalous negative magnetoresistance[1] has been found in Sb-doped Ge and unusually large relaxation time anisotropies $K_\tau=\tau_{11}/\tau_\perp$ were determined from the saturation value of the longitudinal magnetoresistance.[21]

In the literature the conduction in this quasimetallic range has been called impurity band conduction[22] in contrast to hopping conduction or impurity conduction at lower concentrations. Because of their dependence on the dielectric constant and effective masses, on the number of valleys and the central cell potential of the impurity element, the upper and lower concentration limits of these ranges depend on the semiconductor material, the doping element, and the parameters affecting the Bohr radius like stress and magnetic field.

Theoretical studies of Lax and Phillips[23] have shown that the states which lie further than E_D in the band, where E_D is the ionization energy of a single impurity, remain unaffected by the impurities, whereas some of the states which lie lower are pulled down in energy and form a tail to the band which extends into the gap. One might argue that normal metallic behavior is expected only when the Fermi level penetration into the band is larger than E_D, that is $E_F \geq E_D$. This condition is satisfied at $N \geq 1.7\times10^{18}$ for the zero-stress case and at $N \geq 4\times10^{17}$ for case F. Figure 5 shows, however, that for case F the metallic behavior is only approached at concentrations $N \geq 3\times10^{18}$ which are considerably higher than for the zero stress case. Hence other factors seem to play a more important role.

In the following we restrict the analysis to the high-concentration range where the truly metallic behavior appears to be established. The range is unfortunately limited at the upper end by the relatively low solubility of Sb in Ge. The ratio of the mobilities measured in cases G and F yields directly the mobility ratio $K=\mu_\perp/\mu_{11}$. The scatter of data points in Fig. 5 is largely due to the scatter of the zero-stress resistivity as a function of concentration. The saturation values of $\Delta\rho/\rho_x$ as a function of N shown in Fig. 6 fall much better on a smooth curve. The large number of zero-stress data points make it possible to determine μ_0 quite accurately as a function of N. This curve was taken together with the smooth curves of Fig. 6 to determine the dependence of mobility on concentration for the cases C, F, and G. The mobility anisotropy K obtained from measurements G and F is $K=3.9\pm0.1$ for $4\times10^{18}\leq N\leq 9\times10^{18}$. We noticed a slight decrease of K from $K=4.0$ to $K=3.8$ as N increases within this concentration range. This change lies, however, within the error limits.

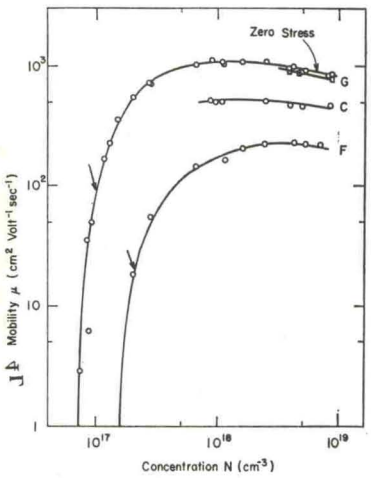

FIG. 5. Mobility components along the current direction determined from zero-stress measurements and the saturation values of the piezoresistance in arrangements C, F, and G. The arrows indicate the critical concentrations N_c at which the transition from nonmetallic to metallic conduction occurs.

Our value $K=3.9\pm0.1$ agrees quite well with the value $K=4.3$ quoted by Koenig[2] for Sb-doped Ge, $N=2.2\times10^{18}$ cm^{-3}. Tsidilkovsky[21] obtained $K=2$ for $N=2.4\times10^{18}$ from the saturation value of the longitudinal magnetoresistance. He finds that K decreases smoothly from $K=6$ to $K=2$ as N increases from 2.5×10^{17} to 2.4×10^{18}, whereas we find a negligible N dependence at somewhat higher N. Although his high-concentration sample lies beyond the range of impurity band conduction it is possible that at high magnetic fields the range limit is shifted to higher concentrations because of the tightening of the electron orbits. His results might therefore be affected by impurity band effects.

We determined K under conditions at which all electrons are in a single lower valley. Since we observed hardly any concentration dependence of K it seems justified to assume that K is quite insensitive to a change in screening and hence to a change in the number of valleys. If this is true one can use $K=3.9$ for the determination of μ_{11} from the mobility components

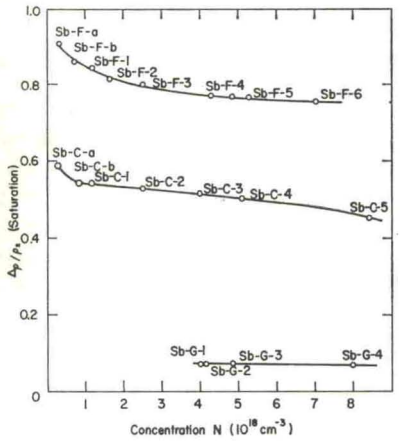

FIG. 6. Saturation values of the piezoresistance for arrangements C, F, and G as a function of carrier concentration.

[21] I. M. Tsidilkovski and V. I. Sokolov, International Conference on the Physics of Semiconductors, Paris, 1964 (unpublished).

[22] Y. Toyozawa, *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Academic Press Ltd., London, 1961), p. 215.

[23] M. Lax and J. C. Phillips, Phys. Rev. **110**, 41 (1958).

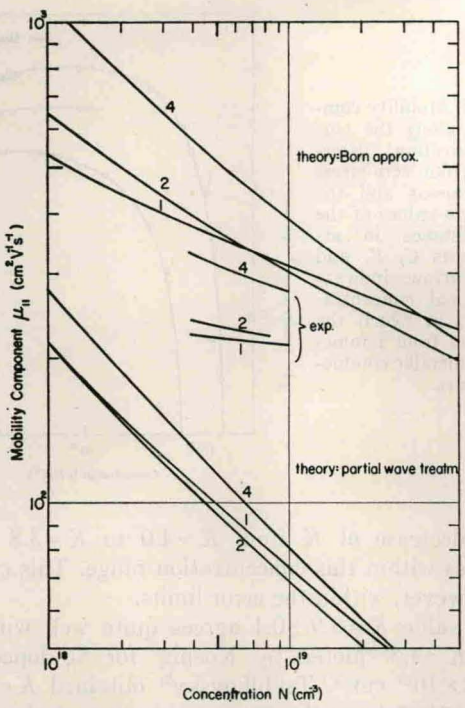

FIG. 7. Mobility component parallel to the valley axis as a function of impurity concentration. The experimental curves and the two sets of theoretical curves are marked on the figure. The numbers are the number of lower valleys.

measured at zero stress and with the C orientation. To check this assumption one sample was measured in the orientation H which is a transverse measurement with two lower valleys at saturation. From the mobility components of samples Sb-C-5 and Sb-H-1 we obtained $K=4.3\pm0.3$ for two lower valleys. This value lies slightly higher. It has, however, a larger uncertainty since the value does not represent an average over a number of measurements. Noting that this problem has not yet been clarified we shall use the assumption of a constant K in the following analysis. From the zero-stress mobility μ_0 and the mobility μ_c of arrangement C the parallel mobility components $\mu_{II}(4)$ and $\mu_{II}(2)$ for four and two lower valleys, respectively, can be determined.

$$\mu_0=\mu_{II}(4)(2K+1)/3\,,\qquad \mu_c=\mu_{II}(2)(K+2)/3\,. \tag{1}$$

The mobility values μ_{II} for one, two, and four lower valleys are shown as a function of N in Fig. 7. One notices that $\mu_{II}(4)$ lies considerably higher than $\mu_{II}(2)$ and $\mu_{II}(1)$. There are two effects that cause the mobility to depend on the number of valleys over which a given concentration of electrons is distributed. (a) The Thomas-Fermi screening radius R decreases with increasing density of states and hence with increasing number of valleys ν

$$R=(\kappa/4\pi e^2 g(E))^{1/2}\,,\qquad g(E)=\text{const}\nu^{2/3}\,, \tag{2}$$

where κ=static dielectric constant and $g(E)$=density of states. This effect indeed yields $\mu_{II}(4)>\mu_{II}(2)>\mu_{II}(1)$. Counteracting this screening effect, however, is the fact (b) that Coulomb scattering is more effective for lower energy than for higher energy carriers. Because $E_F(\nu)=E_F(1)\nu^{-2/3}$ this factor tends to make $\mu_{II}(1)$ larger than $\mu_{II}(4)$. The magnitudes and the relative importance of the two competing effects (a) and (b) can be estimated best by studying the predictions of the theories for ionized impurity scattering in degenerate semiconductors.

B. Ionized Impurity Scattering in the Born Approximation and the Partial-Wave Analysis

All scattering theories available for this problem assume that scattering occurs on N individual scattering centers, each having a total scattering cross section A. The relaxation time τ is then evaluated from

$$\tau=1/vNA, \tag{3}$$

where v is the velocity of the incoming charge carrier. In the present case this assumption is certainly unjustified in view of the fact that the de Broglie wavelength of electrons at the Fermi level and the screening radius R are of the same order of magnitude as the average impurity separation.[2] It is therefore expected that these calculations overestimate the magnitude and the N dependence of the scattering.

Another approximation is made with respect to the scattering anisotropy. Although some attempts[24] have been made to calculate anisotropic impurity scattering in nondegenerate semiconductors, very poor agreement with experiment is obtained. For degenerate multivalley semiconductors the ellipsoidal Fermi surfaces are usually approximated by spheres containing the same number of states.[13] This means the true effective masses are replaced by the density-of-states effective mass $m^*=(m_{II}m_\perp^2)^{1/3}$.

In the following we shall include the scattering anisotropy partly by assuming an isotropic mean free path $\Lambda=1/NA$ and the full anisotropy of the Fermi velocity v_F. This yields a ratio $\tau_{II}/\tau_\perp=(m_{II}/m_\perp)^{1/2}$ and a mobility anisotropy $K=\mu_\perp/\mu_{II}=(m_{II}/m_\perp)^{1/2}$. There is some conflicting evidence concerning the change of effective masses with heavy doping[25] as determined by optical reflectivity studies. Furthermore these measurements determine only the conductivity effective mass $m_\sigma^*=(m_{II}^{-1}+2m_\perp^{-1})^{-1}$. Thus even if m_σ^* is found to be independent of N the mass ratio $m_{II}/m_\perp$ might change. Using the mass ratio of pure germanium one obtains $K=4.4$ in fair agreement with our experimental value $K=3.9\pm0.1$.

[24] F. S. Ham, Phys. Rev. **100**, 1251 (1955); A. G. Samoilovich, I. Ya. Korenblit and I. V. Dakhovskii, Dokl. Akad. Nauk SSSR **139**, 355 (1961) [English transl.: Soviet Phys.—Doklady **6**, 606 (1962)].

[25] For references on this subject see Ref. 5.